ON THE OPTIMIZATION OF REFRIGERATION MACHINERY†

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Abstract. We present the application of thermoeconomics to the optimization of a heat-pump. The method is well suited for application to thermodynamic processes and yields exergy losses. The marginal cost of an arbitrary variable can also be calculated. The efficiencies of the compressor, condenser, evaporator, and electric motor are chosen as variables which are to be optimized. Parameters such as the price of electricity and the temperature of the delivered heat may vary among optimizations, and results are presented for different parameter values. The result shows that the efficiency of the electric motor is most important.

À L'OPTIMISATION D'UN SYSTEME DE REFRIGERATION

RESUME: Cet article présente l'application d'une étude thermique et économique de l'optimisation d'un modèle de cycle de pompe à chaleur. Cette méthode convient bien à l'application aux processus thermodynamiques et donne peu de pertes d'exergie. Le coût marginal d'une variable arbitraire peut aussi être calculé. Les rendements du compresseur, du condenseur, de l'évaporateur et du moteur électrique sont choisis comme les variables de décision à optimiser. Des paramètres tels que le prix de l'éctricité et la température à laquelle la chaleur est fournie peuvent varier suivant les optimisations, les résultats sont présentés pour différentes valeurs des paramétres. Le résultat montre que le rendement du moteur électrique est le plus important.

NOMENCLATURE

a	annuity factor, dimensionless
C	cost per unit time, SEK/yr
E	exergy, J
E_{el}	electricity used per year, J/yr
h	specific enthalpy, J/kg
H	enthalpy, J
k	constant factor
m	mass flow, kg/s
NTU	number of heat transfer units, dimensionless
n_c	quantity of substance c, kg
n_i	depreciation time for component i, yr
P	power, W
p	pressure, Pa
p_{el}	price of electricity, SEK/J
r	interest rate, dimensionless
T	temperature, K
T_0	reference temperature, K
t	operating time per unit time, dimensionless
S	entropy, J/K
V	volume, m^3
x	state parameters
y	optimization or decision variable
z	decision parameters
ΔS^{tot}	total entropy production, J/K
Φ_j	equations of state

†The optimization program is available on diskette by sending USD 10 in cash to the author.

Φ_0	objective function, SEK/yr
η	efficiency, dimensionless
μ	chemical potential, J/kg
θ	marginal cost, SEK/yr

Subscripts

c	substance
el	electricity
i	component
k	refers to optimization variables
r	refrigerant
wc	water on cold side
wh	water on hot side
0	reference state

INTRODUCTION

This study origins from earlier work[1] and the purpose of this paper is to introduce the method of thermoeconomics to refrigeration engineering. The construction of technical systems, which are often based on experience, educated guess-work and personal evaluations, could be improved by an analysis of the effects of different potential solutions in terms of cost. Usually, a maximum cost permitted for each part of the system is set, and the market prices determine to what extent efficient components can be afforded. Such systems always cost at least as much as and often more than they would if thermoeconomic optimization were used.

El-Sayed and Tribus[2] developed the concept of thermoeconomics, in which the objective function is optimized, subject to given economic and technical constraints. The purpose of thermoeconomics is to improve analyses of systems by introducing ways of concurrently suggesting improvements.

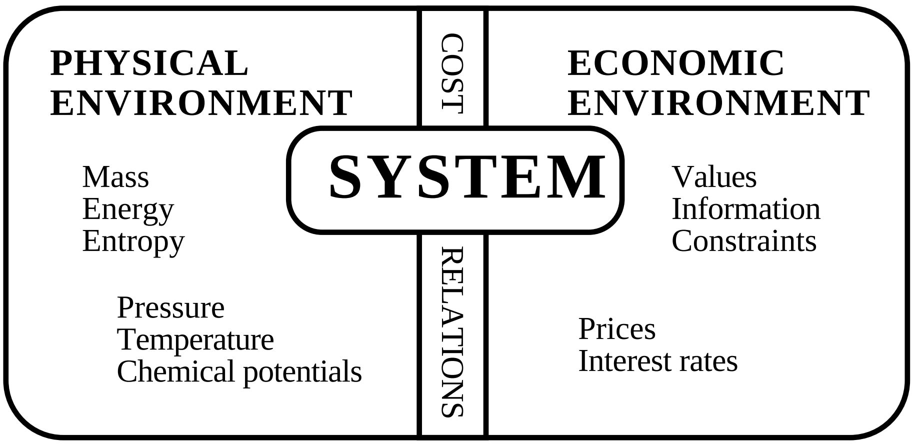

Fig. 1. The system in a physical and an economic environment.

By optimizing the total cost of a system in operation, we always find the best system within the given physical and economic conditions. We can also calculate the marginal costs of the exergy losses in each component. These values are very important in the selection of research and development measures, or in the improvement of an existing system.

The system is described in relation to the physical (pressure, temperature, and chemical potentials of the appropriate substances) and economic environments (prices of goods and prices of capital or interest rates). These two environments are interrelated by cost relations based on physical quantities, see Fig. 1.

With regard to the physical environment, the energy and mass flows are evaluated in physical terms, i.e., in terms of exergy per unit time. The difference between all incoming exergy flows and all outgoing exergy flows must be minimized and the efficiency must be maximized. In the economic environment all energy and mass flows are evaluated instead in terms of economic value or costs. The main function is now the cost per unit time, (i.e., operation and capital costs minus income), which should be minimal. Thermoeconomic optimization is an economic optimization subject to the physical constrains of the system.

THERMOECONOMIC OPTIMIZATION

The objective function Φ_0 is defined as a function of state parameters $\{x_j\}$, where $\{x_j\}$ is abbreviation for $x_1, x_2,\ldots x_j\ldots x_n$, decision variables $\{y_k\}$, and decision parameters $\{z_l\}$, i.e.

$$\Phi_0 = \Phi_0 (\{x_j\},\{y_k\},\{z_l\}), \tag{1}$$

where j=1, 2,…, n, k=1, 2,…, m, and l=1, 2,…, r.

The equations of state are

$$\Phi_j (\{x_i\},\{y_k\},\{z_l\}) = 0, \qquad j = 1, 2,\ldots, n. \tag{2}$$

The optimization is formulated as follows:

$$\text{minimize } \Phi_0 = \Phi_0 (\{x_i\},\{y_k\},\{z_l\}), \tag{3}$$

$$\text{subject to } \Phi_j (\{x_i\},\{y_k\},\{z_l\}) = 0, \qquad j = 1, 2,\ldots, n. \tag{4}$$

The exergy losses due to irreversibilities in a stationary state is determined for each component by regarding in and outflows of exergy. The exergy content is

$$E = H - T_0 S - \sum_c \mu_{c0} n_c . \tag{5}$$

For the system, we obtain a sum for all components, which gives the total rate of exergy loss. This may also be written as the product of the reference temperature and total entropy production, i.e., $T_0\,\Delta S^{tot}$.

REFRIGERATION MACHINERY

Heat pump systems offer much more efficient means of producing heat than traditional combustion or electrical short circuit technologies. Heat pump systems are therefore becoming more common as the prices of fuels and electricity increase. The configuration of the system in this study is defined in Fig. 2. It consists of a compressor, a condenser, an expansion valve, an evaporator, and an electric motor.

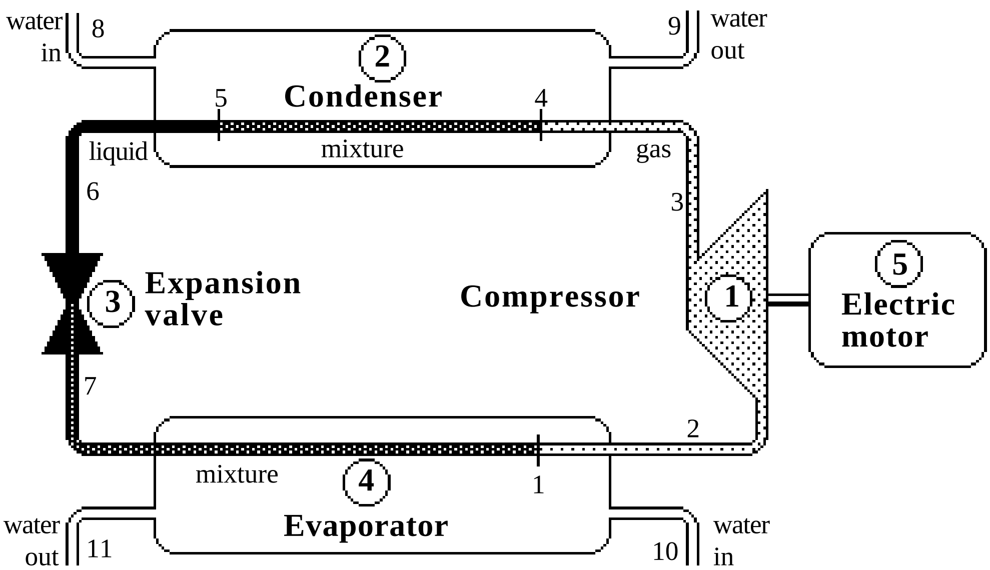

Fig. 2. Heat pump system with 5 components and 11 flows.

The refrigerant is superheated after passing through the evaporator, step 1-2, and supercooled after passing through the condenser, step 5-6. The actual state of the refrigerant after the compression, 3, differs from that of a reversible process, 3_{rev}, due to the limited efficiency of the compressor. The heat produced from the system is $h_3 - h_6$, the heat input is $h_2 - h_7$, and the work supplied to the compressor is $h_3 - h_2$. The electricity input required to operate the system becomes $(h_3 - h_2)/\eta_5$, where η_5 is the efficiency of the electric motor.

The system is completely defined apart from the decision variables $\{y_k\}$ which are the efficiencies of the compressor, the condenser, the evaporator, and the electric motor. These are defined as follows

$$y_1 = \eta_1 = \frac{h_{3rev} - h_2}{h_3 - h_2}, \tag{6}$$

$$y_2 = \eta_2 = \frac{T_9 - T_8}{T_4 - T_8}, \tag{7}$$

$$y_3 = \eta_4 = \frac{T_{11} - T_{10}}{T_1 - T_{10}}, \tag{8}$$

$$y_4 = \eta_5 = \frac{m_r(h_3 - h_2)}{P} \quad . \tag{9}$$

Each set of these variables determines a state of the system. The exergy flows and exergy losses can now easily be determined for each component.

The objective is to minimize the total life cycle cost, which includes both the operating (electricity) cost and the capital cost, for a given amount of produced heat. The operating cost increases if the investments decrease and vice versa. For the designer the market value of the product (heat) and a given required profit sets an upper limit for the total cost of the system. The problem is to split this cost between the operating cost and the capital cost for each component. (The costs for parts slightly affected by alternative constructions of the system, such as pipes connecting the components, are just added as constants since they have no effect on the optimization.) The following assumptions, concerning the size of and external conditions on the system, are made: heat produced 6.5 kW (energy power), operation time/yr 5000 hr, price of electricity SEK 0.25/kWh (SEK 6 ≈ US$ 1), and temperature of the produced heat (T_9) 60°C. Let us assume the following simple cost relations† for the 5 components to be valid in the region of optimization for this system as defined below.[2]

Compressor
$$C_1 = a_1 k_1 \frac{V_2}{0.9 - \eta_1} \frac{p_3}{p_2} \ln \frac{p_3}{p_2} \tag{10}$$

Condenser
$$C_2 = a_2 k_2 m_{wh} \sqrt{\frac{\eta_2}{1-\eta_2}} = a_2 k_2 m_{wh} \sqrt{e^{NTU_2} - 1}\,, \tag{11}$$

Expansion Valve
$$C_3 = a_3\, k_3\, m_r\,, \tag{12}$$

Evaporator
$$C_4 = a_4 k_4 m_{wc} \sqrt{\frac{\eta_4}{1-\eta_4}} = a_4 k_4 m_{wc} \sqrt{\mathrm{e}^{NTU_4} - 1})\,, \tag{13}$$

Electric Motor
$$C_5 = a_5 k_5 P \frac{\eta_5}{1-\eta_5}\,. \tag{14}$$

Furthermore the annuity factors of the different capital investments are defined as

$$a_i = \frac{r}{1-(1+r)^{-n_i}}\,. \tag{15}$$

The depreciation time n_i may vary for each component due to variations in economic lifetime and maintenance costs such as renovations, etc.

Figure 3 shows the investment costs as a function of the efficiencies. A typical “knee”, from the “penalty function”, can be observed for the compressor and the electric motor at specific values of the efficiencies, approximately at 86 and 94 % respectively. If the compressor and the electric motor are regarded as given, or limited to a number of possibilities, the decision space is limited to one, or a number of, two dimensional rooms, defined by the efficiencies of the condenser and the evaporator, i.e., the only two decision variables.

† These relations may be changed due to other assumptions of the system, e.g., non linear relations to the size of the system.

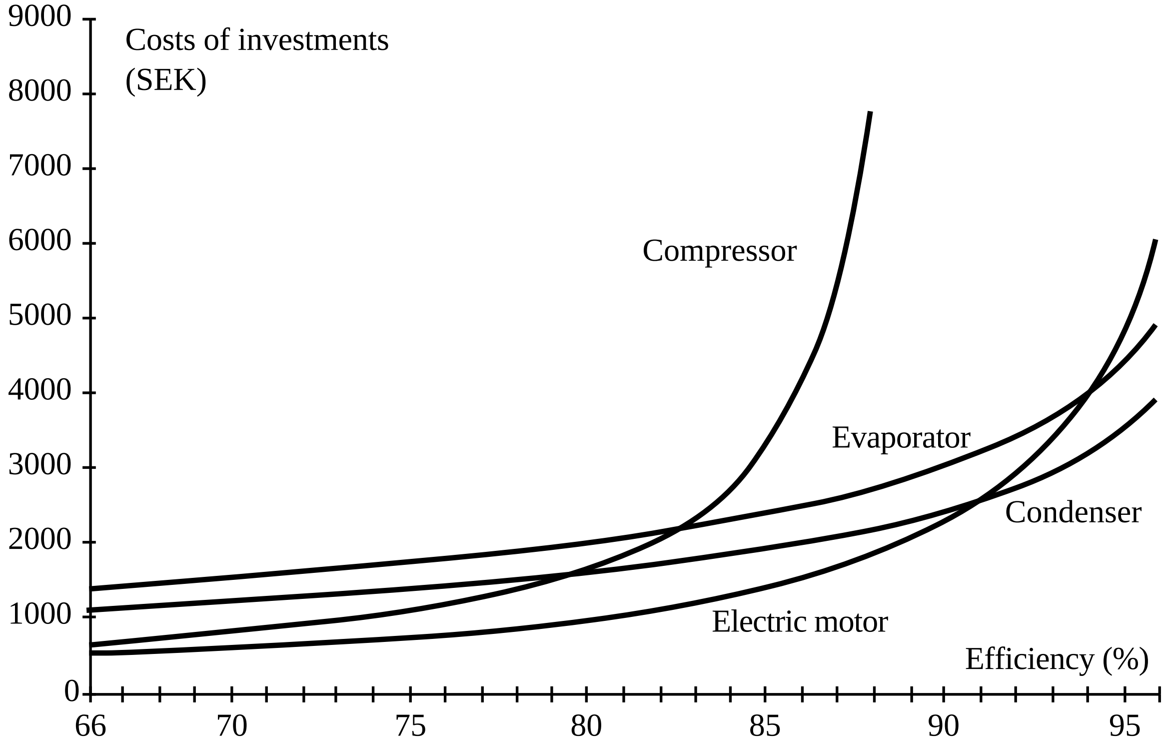

Fig. 3. Costs of investments as a function of the efficiencies.

The investment costs are depreciated according to the annuity method, which gives a cost per unit time for every component. The total cost per unit time, i.e., the objective function, is the sum of these costs and the cost of the electricity used, during the same period of time,

$$\Phi_0 = \sum_{i=1}^{5} C_i + tp_{el}E_{el}. \tag{16}$$

RESULTS AND CONCLUSIONS

The optimization is achieved by calculating the value of the objective function Φ_0 and the marginal costs $\{\theta_k\}$ for every set of the variables $\{y_k\}$ according to

$$\theta_k = \frac{\Delta\Phi_0}{\Delta y_k}, \qquad k = 1, 2, 3, 4. \tag{17}$$

From these values a new set of variables $\{y_k\}$ is determined by using the Newton-Raphson method. Thus the system moves towards the nearest minimum from the given start values. However, the problem is strongly non-linear which means that there is no general method for finding the global minimum. Instead common sense and insight into how the system works must be used to determine the value of a solution.

A computer program has been developed for finding the optimum system. The calculation of the thermodynamic data, for the assumed refrigerant R12†, are based on similar computer-based calculations by Reynolds.[3] The equations of state for the system are formulated so as to avoid iterations. The actual minimizing procedure is carried out with a small number of iterations. When the sum of the marginal costs θ_k is less than a predefined value the optimization is completed.

Let us assume a system with the following values for the efficiencies of the compressor, the condenser, the evaporator, and the electric motor, i. e., the decision variables, 0.7, 0.8, 0.7, and 0.75.‡

† Other refrigerants may also be used by simple changes in the program.

‡ These roughly correspond to real values of a system of this size.

The total cost then becomes SEK 3676/yr of which SEK 3025/yr relate to electricity. From the optimization we get the following values of the efficiencies: compressor 0.80, condenser 0.82, evaporator 0.72 and electric motor 0.91. The total cost now amounts to SEK 3167/yr of which SEK 2237/yr relate to electricity. By increasing the investments from SEK 651/yr to SEK 930/yr, the total cost of the system becomes SEK 509/yr (about 14%) less than for the assumed system, see left side of Fig. 4. At the same time the exergy losses decrease from 1464 W to 839 W, i.e., by 625 W (T_0 = 0˚C). From Fig. 4 we also see that it is the improvement of the electric motor that gives the largest single exergy saving. Thus, the optimization in this case saves both money and exergy.

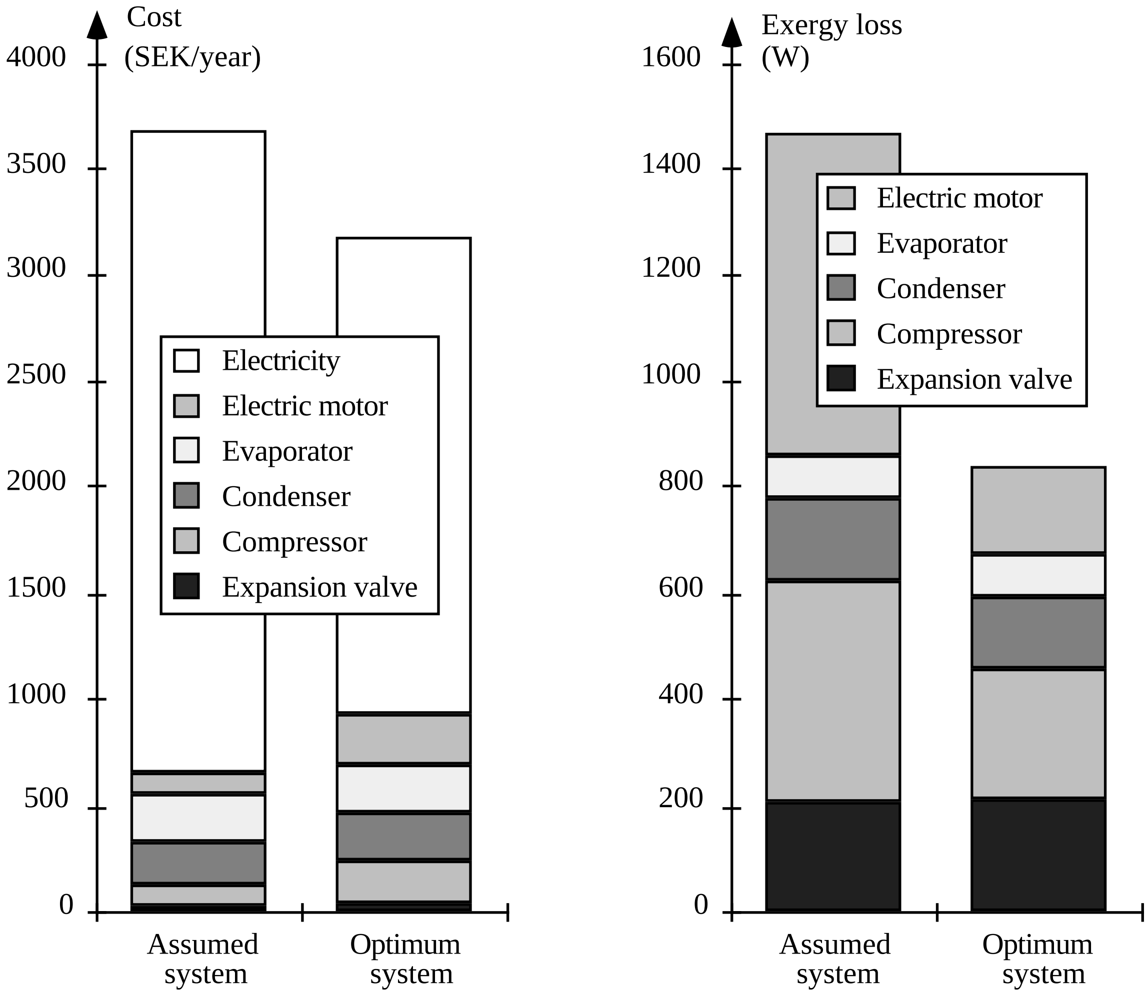

Fig. 4. Costs and exergy losses for the assumed and the optimum systems.

The expansion valve accounts for a larger fraction of the total exergy loss in the optimum system, which further justifies investment in research and development to improve it.

The result indicates the electric motor to be the most critical component to improve. The electric motor is assumed to cost approximately 2.4 times as much at 91% efficiency than at 75% efficiency, which must be regarded as realistic. (But, it may even cost up to 7.7 times as much and still be competitive with the assumed system.) It may also be added that the coefficient of performance (COP) increases from 2.69 for the assumed system to 3.63 for the optimum system.

In order to further show the usefulness of the method the dependence on or sensitivity to the cost of electricity and the temperature of the produced heat (T_9) has also been studied. Figure 5, 6, and 7 shows the relationship between the component costs and the price of electricity, when this varies between SEK 0.2 and 2/kWh, when the temperature of the produced heat is 50, 60, or 70˚C. At 50˚C (T_9) the total cost increases from SEK 2296/yr to SEK 13891/yr at SEK 2/kWh. This can be seen in relation to the fact that if the optimum system at SEK 0.2/kWh had been used at SEK 2/kWh, then the total cost would be SEK 15913, i.e., an increase in the cost of SEK 2022/yr. (For the assumed system the total cost would instead be SEK 24851, i.e., a cost increase of SEK 10960/yr.) From Figs. 5-7 we see that all components should become more expensive, thus more efficient, when the price of electricity increases. This might have been anticipated, but the exact interrelations could not have been foreseen.

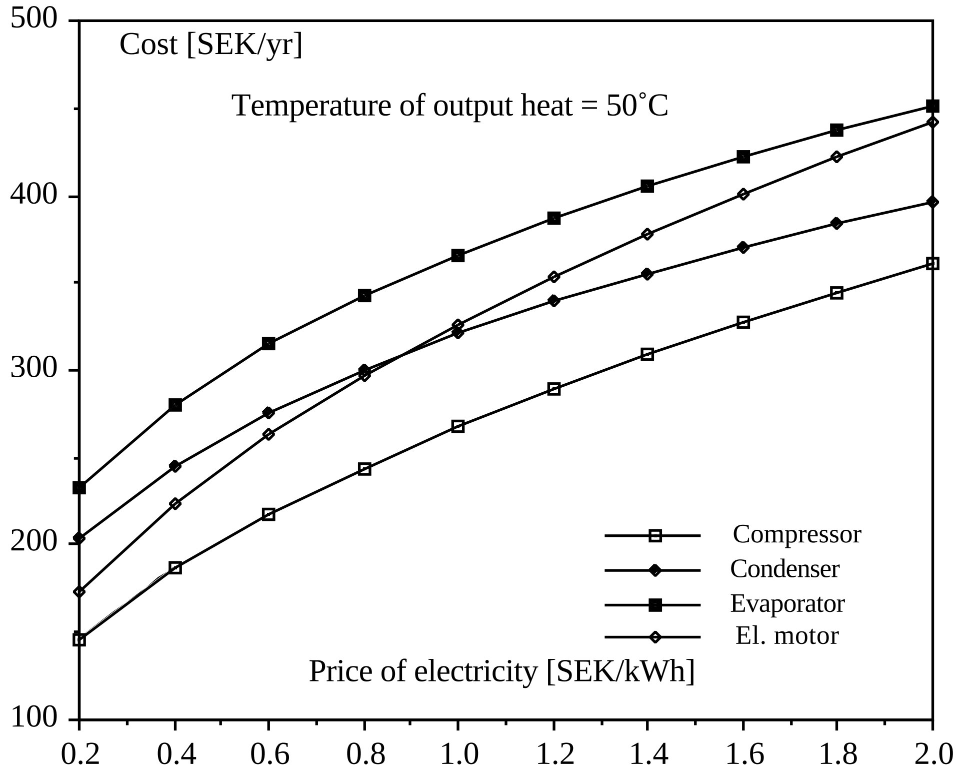

Fig. 5. Component costs as a function of the price of electricity when output heat is 50˚C.

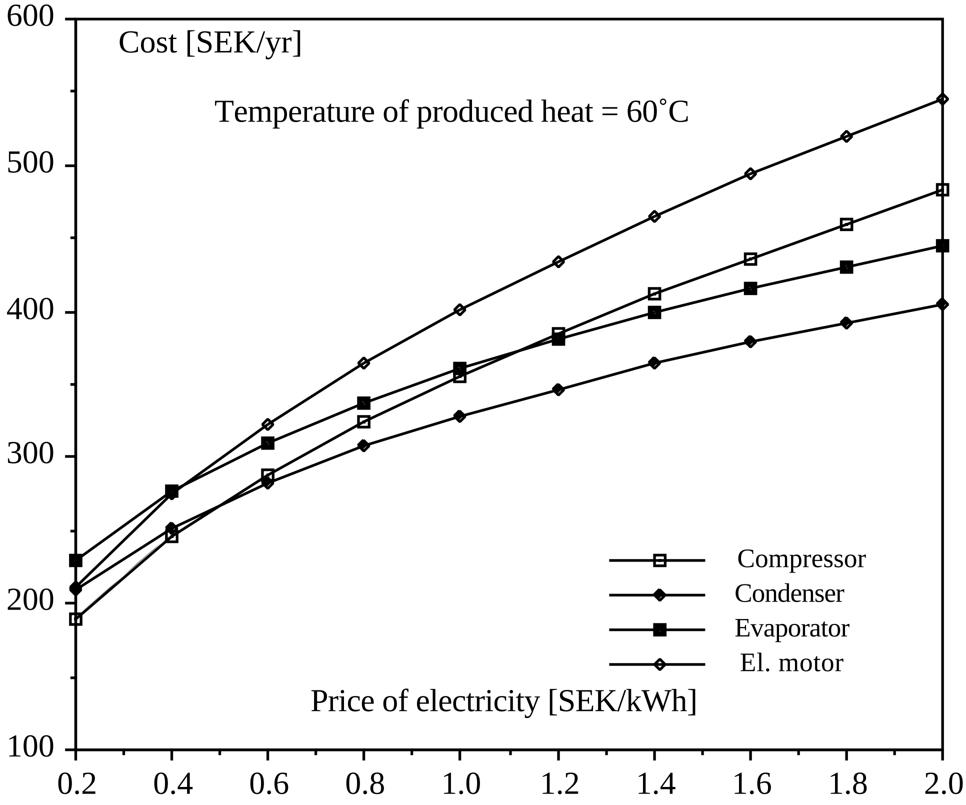

Fig. 6. Component costs as a function of the price of electricity when output heat is 60˚C.

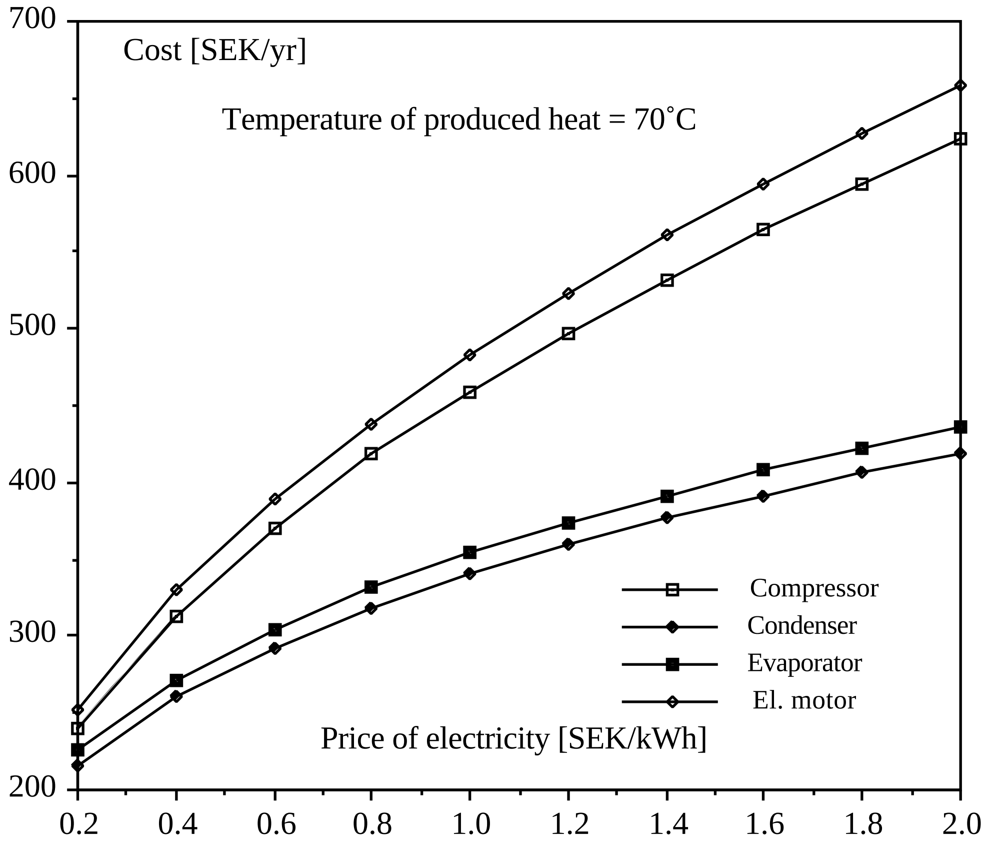

Fig. 7. Component costs as a function of the price of electricity when output heat is 70˚C.

When the temperature of the produced heat (T_9), is changed from 40 to 80˚C, at an electricity price of SEK 0.25/kWh, the total cost increases from 2203 to SEK 4333/yr and the COP changes from 5.6 to 2.6. The heat produced (energy) is 6500 W, but the rate of exergy changes from 434 to 799 W (T_0 = 0˚C) which better explains the increased cost. When the temperature increases from 40 to 80°C the total system, but not necessarily each component, must be more efficient, see Fig. 8. Within a total increase of component costs and efficiencies, we see that, it is more economical to choose a less expensive evaporator. The explanation is simply that investments in other parts of the system pay off better.

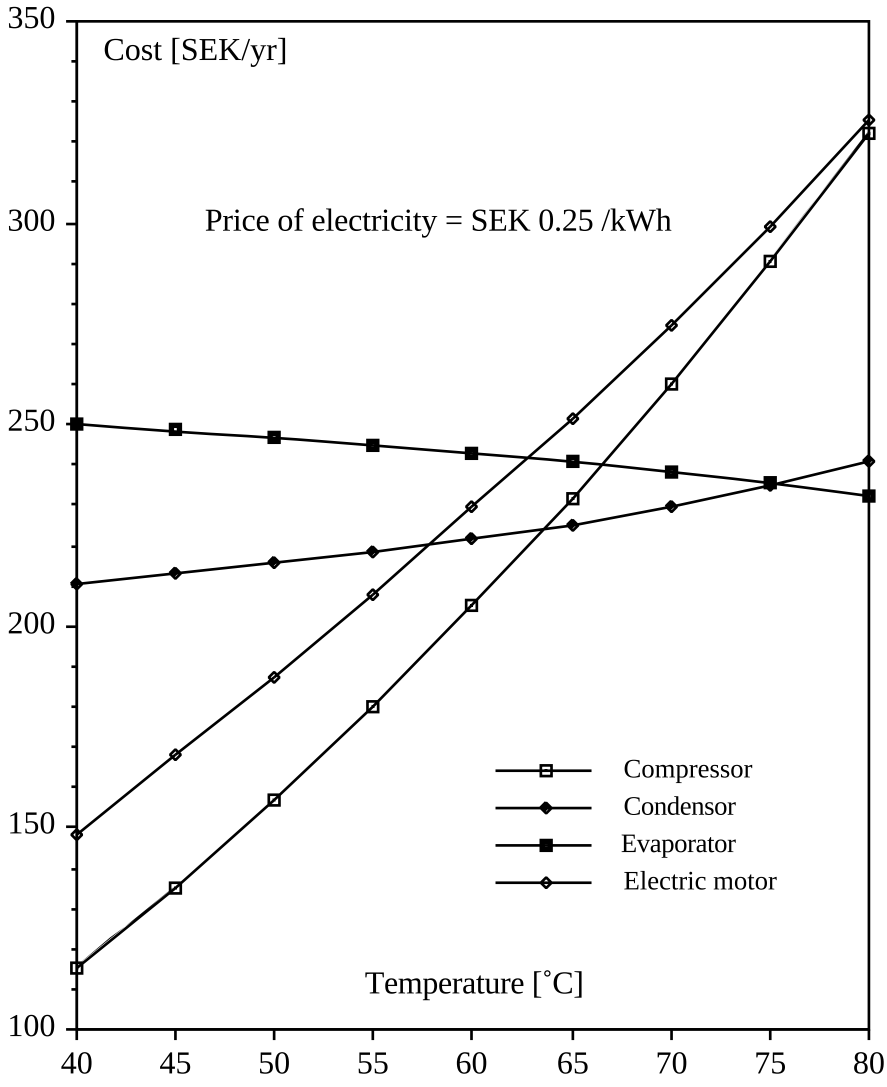

Fig. 8. Component costs as a function of the temperature of the produced heat.

Similarly, many other relations may be described by using the computer program for the system. The program can easily be rewritten for other refrigerants or cost relations. The purpose of this study is merely to show the gain achieved by applying the method of thermoeconomics to an example of a refrigeration machinery. The exact numerical results for describing thermoeconomics as a method for improving technical processes have therefore been neglected.

It must be noted that thermoeconomics can never replace long experience and high technical and economic competence. Manufacturers will always be driven by two strong driving concerns: minimum cost and minimum performance standards. However, if they considered the system as a way to produce heat they should offer *minimum cost of heat* rather than minimum cost of poor equipment. In any case, thermoeconomics will always be an important complementary tool.

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